International Journal of Theoretical Physics, Vol. 44, No. 9, September 2005 (© 2005)
DOI: 10.1007/s10773-005-4771-1

Area Spectrum of Near Extremal Black Branes from Quasi-Normal Modes

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Received November 27, 2004; accepted March 30, 2005

Bekenstein and others propose that the black hole area spectrum is discrete and equally spaced. We implement Kunstatter's method to derive the area spectrum for near extremal black 3-branes. The area spectrum of the event horizon is discrete but not equally spaced.

KEY WORDS: area spectrum; quasi-normal modes; black brane; Bekenstein–Hawking entropy.

1. INTRODUCTION

Dynamical properties of a thermal gauge theory are encoded in its Green's functions. In the context of AdS/CFT (Maldacena, 1998), Minkowski-space Green's functions can be computed from gravity using the recipe given in Son and Starinets (2002). Unfortunately, for a non-extremal background, only approximate expressions for the correlators are usually obtained. Even thought the retarded Green's function in four dimensions cannot be found explicitly, the location of its singularities can be determined precisely. As shown by Son and Starinets (2002), this amounts to finding the quasi-normal frequencies of dilaton fluctuation in the dual near extremal black brane background as functions of the spatial momentum. The possibility of a connection between the quasi-normal frequencies of black holes and the quantum properties of the entropy spectrum was first observed by Bekenstein (1997), and further developed by Hod (1998). Bekenstein noted that Bohr's correspondence principle implies that frequencies characterizing transitions between energy levels of a quantum black hole at large quantum numbers correspond to the black hole's classical oscillation (quasi-normal mode) frequencies (see also Kokkotas and Schmidt, 1999; Nollert, 1999). In particular, Hod proposed that the real part of the quasi-normal frequencies, in the infinite damping

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limit (i.e., the $n \to \infty$ limit), might be related *via* Bohr's correspondence principle to the fundamental quanta of mass and angular momentum (see also Abdalla *et al.*, 2003; Bekenstein, 2000; Bekenstein and Mukhanov, 1995; Birmingham, 2003; Cardoso and Lemos, 2003; Corichi, 2003; Kastrup, 1996; Padmanabhan, 2004; Padmanabhan and Patel, 2003; Polychronakos, 2003; Roy Choudhury and Padmanabhan, 2004).

In asymptotically flat spacetimes, the idea of QNMs started with the work of Regge and Wheeler (1957) where the stability of a Schwarzschild black hole was tested, and were first numerically computed by Chandrasekhar and Detweiler (1975) several years later. Recently, there has been considerable interest in studying the quasi-normal modes in different contexts: in AdS/CFT duality conjecture (Birmingham and Carlip, 2004; Birmingham *et al.*, 2002; Cardoso and Lemos, 2001a,b; Chan and Mann, 1997; Horowitz and Hubeny, 2000; Kim and Oh, 2001; Moss and Norman, 2002; Wang *et al.*, 2000), when considering thermodynamic properties of black holes in loop quantum gravity (Dreyer, 2003; Kunstatter, 2003; Ling and Zhang, 2003), in the context of possible connection with critical collapse (Horowitz and Hubeny, 2000; Kim and Oh, 2001; Konoplya, 2002), also when considering the area spectrum of black holes (Das and Shankaranarayanan, 2004; Lepe and Saavedra, 2005; Setare, 2004a,b; Setare and Vagenas, 2004). Recently, it has been observed that the quasi-normal modes can play a fundamental role in loop quantum gravity (Dreyer, 2003). Dreyer showed that in order to have consistency between the Bekenstein–Hawking entropy calculation and quasi-normal-mode frequencies, one had to assume that the minimum of j of the spin network piercing the horizon and contributing significantly to the entropy had to be $j = 1$. With this choice, the resulting Immrizi parameter would be given by $\gamma = \frac{\ln 3}{2\pi\pi\sqrt{2}}$. He suggested that if the gauge group of the theory were changed from $SU(2)$ to $SO(3)$ then this requirement would be immediately satisfied.

For the Schwarzschild black hole in four dimensions, the asymptotic real part of the gravitational quasi-normal frequencies is of the form $\omega = T_{\mathrm{H}} \ln 3$ where T_{H} is the Hawking temperature (Motl, 2003). The suggestion of Hod was to identify $\hbar\omega$ with the fundamental quantum of mass ΔM. This identification immediately leads to an area spacing of the form $\Delta A = 4\,\hbar \ln 3$. An elegant approach, for the Schwarzschild black hole in d dimensions, based on analytic continuation and computation of the monodromy of the perturbation was proposed in (Motl and Neitzke, 2003).

In the present paper, we extend directly Kunstatter's (2003) approach to determine mass and area spectrum of the near extremal black 3-branes. According to this approach, an adiabatic invariant $I = \int \frac{dE}{\omega(E)}$, where E is the energy of system and $\omega(E)$ is the vibrational frequency, has an equally spaced spectrum, i.e., $I \approx n\,\hbar$, applying the Bohr–Sommerfeld quantization at the large n limit.

2. COINCIDENT D-3 BRANES

We consider now the background (in the string frame) of a black hole describing a number of coinciding D-3 branes (Klebanov, 1999; Kiritsis, 1999)

$$ds^2 = H^{-1/2}(r)\left[-f(r)dt^2 + \sum_{i=1}^{3}(dx^i)^2\right] + H^{1/2}\left[f^{-1}(r)dr^2 + r^2 d\Omega_5^2\right], \quad (1)$$

where

$$H(r) = 1 + \frac{l^4}{r^4}, \quad f(r) = 1 - \frac{r_0^4}{r^4}, \quad (2)$$

and $d\Omega_5^2$ is the metric of a unit five-dimensional sphere. The horizon is located at $r = r_0$ and the extremality is achieved in the limit $r_0 \to 0$. For l much larger than the string scale $\sqrt{\alpha'}$, the entire 3-brane geometry has small curvatures every where and is appropriately described by the supergravity approximation to type IIB string theory (Klebanov, 1999). The requirement $l \gg \sqrt{\alpha'}$ translates into the language of $U(N)$ SYM theory on N coincident D-3 branes. To this end, it is convenient to equate the ADM tension of the extremal 3-brane classical solution to N times the tension of a single D-3 brane. Then one can find (Gubser *et al.*, 1996)

$$\frac{2l^4\Omega_5}{k^2} = N\frac{\sqrt{\pi}}{k}, \quad (3)$$

where $\Omega_5 = \pi^3$ is the volume of a unit 5 sphere, and $k = \sqrt{8\pi G}$ is the 10-dimensional gravitational constant. Therefore,

$$l^4 = \frac{kN}{2\pi^{5/2}}. \quad (4)$$

In the other hand, we have

$$k = 8\pi^{7/2} g_s \alpha'^2, \quad (5)$$

where g_s is the string coupling, then we obtain

$$l^4 = 4\pi N g_s \alpha'^2. \quad (6)$$

The parameters l and r_0 are related to the ADM mass in the following way

$$M = \frac{\Omega_5 V_3}{2k_{10}^2}\left(5r_0^4 + 4l^4\right). \quad (7)$$

Now we would like to consider the near extremal 3-brane geometry. In the near-horizon region, $r \ll l$, we may replace $H(r)$ by $\frac{l^4}{r^4}$. The resulting metric is as following

$$ds^2 = \frac{r^2}{l^2}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \frac{l^2}{r^2}\left(1-\frac{r_0^4}{r^4}\right)^{-1} dr^2 + l^2 d\Omega_5^2. \quad (8)$$

The metric given earlier is a product of S^5 with a certain limit of the Schwarzschild black hole in AdS_5. The eight-dimensional area of the horizon can be read off from metric (8). If the spatial volume of the D-3 brane is taken to be V_3, then we find

$$A_{\mathrm{h}} = \left(\frac{r_0}{l}\right)^3 V_3 l^5 \Omega_5 = \pi^6 l^8 T^3 V_3, \quad (9)$$

where T is a temperature

$$T = \frac{r_0}{\pi l^2}. \quad (10)$$

Using (4) we arrive at the Bekenstein–Hawking entropy (Gubser *et al.*, 1996)

$$S_{\mathrm{BH}} = \frac{2\pi A_{\mathrm{h}}}{k^2} = \frac{\pi^2 N^2 V_3 T^3}{2}. \quad (11)$$

3. QUASI-NORMAL MODES AND AREA SPECTRUM

Given a system with energy E and vibrational frequency $\omega(E)$, one can show that the quantity

$$I = \int \frac{dE}{\omega(E)} \quad (12)$$

where $dE = dM$, is an adiabatic invariant (Kunstatter, 2003) and as already mentioned in Section 1, *via* Bohr–Sommerfeld quantization has an equally spaced spectrum in the large n limit

$$I \approx n\hbar. \quad (13)$$

The large n asymptotic behavior of quasi-normal frequencies given by following expression (Starinets, 2002)

$$\omega_n^{\pm} = \omega_0^{\pm} \pm 2\pi T n(1 \mp i), \quad (14)$$

where

$$\omega_0^{\pm} = \pi T \lambda_0^{\pm}, \quad (15)$$

where $\lambda_0^{\pm} \approx \pm 1.2139 - 0.7775i$. Now by taking ω_R as

$$\omega_R = \pm\pi T(2n + 1.2139), \quad (16)$$

then by substituting Equation (10) we get

$$\omega_R = \frac{\pm r_0}{l^2}(2n + 1.2139). \tag{17}$$

By taking M as given by Equation (7) and substituting Equation (6), we obtain

$$M = \frac{V_3}{128\pi^4\alpha'^4 g_s^2}\left(5r_0^4 + 16\pi N g_s \alpha'^2\right). \tag{18}$$

Then, the parameter r_0 is given by

$$r_0 = (aM - b)^{1/4}, \tag{19}$$

where

$$a = \frac{128\pi^4\alpha'^4 g_s^2}{5V_3}, \quad b = \frac{16\pi N\alpha'^2 g_s}{5}. \tag{20}$$

Now by taking ω_R as given by expression (17) and substituting Equations (19) and (6), we get

$$\omega_R = \frac{\pm(aM - b)^{1/4}}{2\alpha'\sqrt{\pi N g_s}}(2n + 1.2139). \tag{21}$$

Thus, the adiabatically invariant integral (12) is written as

$$I = \frac{\pm 2\alpha'\sqrt{\pi N g_s}}{(2n + 1.2139)} \int \frac{dM}{(aM - b)^{1/4}}, \tag{22}$$

and after integration, we obtain

$$I = \frac{5V_3\sqrt{N}}{48\pi^{7/2}\alpha'^3 g_s^{3/2}(2n + 1.2139)}(aM - b)^{3/4}. \tag{23}$$

Now using Equations (10) and (19), we can rewrite the area of the horizon Equation (9) as

$$A_h = \pi^3 l^2 V_3 r_0^3 = \pi^3 l^2 V_3 (aM - b)^{3/4}. \tag{24}$$

Using Equation (23) the area is given by following expression

$$A_h = \frac{3(2n + 1.2139)}{160}\pi^7\alpha'^4 g_s^2 I = \frac{3}{160}\pi^7\alpha'^4 g_s^2(2n + 1.2139)n\,\hbar. \tag{25}$$

It is obvious that the area spectrum, although discrete, is not equivalently spaced. The quasi-normal frequencies, which given by Equation (14) were later generalized in the paper by Nunez and Starinets (2003, Equations (3.22)–(3.23)), but all these formulae are asymptotics rather than exact results. However, for vector perturbations (and spatial momentum on the brane equal to zero) the spectrum turns out to be exact and given by following relation

$$\omega_n = n(1 - i), \quad n = 0, 1, \ldots \tag{26}$$

therefore, the integral equation (12) yields

$$I = \frac{M}{n}, \tag{27}$$

and by equating expressions (27) and (13), we get

$$M = n^2 \hbar. \tag{28}$$

It is obvious that the mass spectrum of 3-black brane is quantize.

4. CONCLUSION

The possibility of a connection between quasi-normal modes and the area spectrum of black holes has been actively pursued over the past year. Many examples have been studied, and progress has been made towards a general understanding of this connection. Bekenstein's idea for quantizing a black hole is based on the fact that its horizon area, in the non-extremal case, behaves as a classical adiabatic invariant. It is interesting to investigate how near extremal black 3-branes would be quantized. Discrete spectra arise in quantum mechanics in the presence of a periodicity in the classical system, which in turn leads to the existence of an adiabatic invariant or action variable. Bohr–Sommerfeld quantization implies that this adiabatic invariant has an equally spaced spectrum in the semi-classical limit. Kunstatter showed that this approach works for the Schwarzschild black holes in any dimension, giving asymptotically equally spaced areas. Previously, we have showed that the generalization to non-rotating BTZ, extremal Reissner–Nordström, near extremal Schwarzschild–de Sitter Kerr and extremal Kerr black holes (Lepe and Saavedra, 2005; Setare, 2004a,b,c; Setare and Vagenas, 2004) is also successful.

In this article, we have considered the near extremal black 3-branes. Using the results for highly damped quasi-normal modes in Equation (14), we obtained the area spectrum of event horizon in Equation (25). It is obvious that the area spectrum, although discrete, is not equally spaced. Using the generalized form of quasi-normal modes Equation (26), we obtained the mass spectrum of the near extremal 3-black brane as Equation (28). A similar situation occurs for a BTZ black hole (Setare, 2004a,b), where the area of the event horizon is not equally spaced, in contrast with area spectrum of the black hole in higher dimensions, although the mass spectrum is equally spaced.

ACKNOWLEDGMENT

I would like to thank Prof. Andrei O. Starinets who introduced me the references (Klebanov, 1999; Kiritsis, 1999; Nunez and Starinets, 2003).

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